Effect of Quantum Point Contact Measurement on Electron Spin State in Quantum Dot

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(Dated: January 9, 2009)

We study the time evolution of two electron spin states in a double quantum-dot system, which includes a nearby quantum point contact (QPC) as a measurement device. We obtain that the QPC measurement induced decoherence is in time scales of microsecond. We also find that the enhanced QPC measurement will trap the system in its initial spin states, which is consistent with quantum Zeno effect.

PACS numbers: 03.67.Lx, 03.65.Yz, 03.65.Ta

Introduction. Recently, electron spin in semiconductor quantum dot becomes one of the most promising candidates for quantum computing [1, 2]. The single electron spin states [1] or singlet and triplet states of two electrons in double quantum-dot [3] have been proposed as qubits in quantum computing. However these spin qubits are not completely decoupled from the environment, and there are various sources of decoherence that are intrinsic to the quantum dot system. For example, the electrons spins couple to phonon in the surrounding lattice or other fluctuations via the spin-orbit interaction [4, 5, 6] and the electron spins couple to the surrounding nuclei via the hyperfine interaction [7, 8]. The spin-orbit related relaxation time $T_1 \approx 1$ ms has been demonstrated experimentally [9]. Recently through spin-echo [10] or dynamical nuclear polarization technology [11] to suppress the influence from nuclear spins, the dephasing time $T_2 \approx 1$ μs is also achieved in double quantum-dot system.

On the other hand, as a read-out device, quantum point contact (QPC) is used as a detector [2] to read out spin information in quantum dot system, meanwhile it has back action to the system [12, 13]. It is naturally desired to study how QPC affects the spin states of the quantum dot via Coulomb interaction during the measurement process. It is anticipated that the decoherence in the measurement process would be especially important in case the influence of nuclear spin is suppressed

[10, 11] or there is completed eliminated hyperfine interaction in the SiGe or graphene quantum dots [14, 15].

In this paper, firstly, we introduce a model for two electron spins in a double quantum-dot system with a nearby QPC, give the master equation and calculate the spin states evolution due to the QPC measurement. We make some numerical predictions of QPC induced decoherence time for real experimental parameters. Further, we find that the enhanced QPC measurement will slow down the transition rate between two spin states, which is consistent with quantum Zeno effect.

Model and master equation. Our model is motivated by recent experiment for manipulation and measurement of two electron spin states in a double quantum-dot system [10]. As shown in Fig. 1a, a double-dot system is formed by a layer of two dimensional electron gas restrained by several electrostatic gates used to control the potentials of individual dots and the inter-dot tunneling. Its low energy spectrum is plotted in Fig. 1b. In the measurement process, the double quantum-dot system is set in the biased regime (gray area in Fig. 1b) which can be reduced to an artificial three-level system with the Hamiltonian

$$
\begin{aligned}
\mathcal{H}_0 = {} & E_T \left|(1,1)T_0\right\rangle \left\langle(1,1)T_0\right| + E_S \left|(1,1)S\right\rangle \left\langle(1,1)S\right| \\
& - \varepsilon \left|(0,2)S\right\rangle \left\langle(0,2)S\right| + T_c(\left|(1,1)S\right\rangle \left\langle(0,2)S\right| \\
& + \left|(0,2)S\right\rangle \left\langle(1,1)S\right|),
\end{aligned} \tag{1}
$$

where the notation (n_l, n_r) indicates n_l electrons on the "left" dot and n_r electrons on the "right" dot, S and T represent spin singlet and triplet states, ε and T_c denote the external voltage bias and tunneling amplitude between two dots. It is a good approximation to set
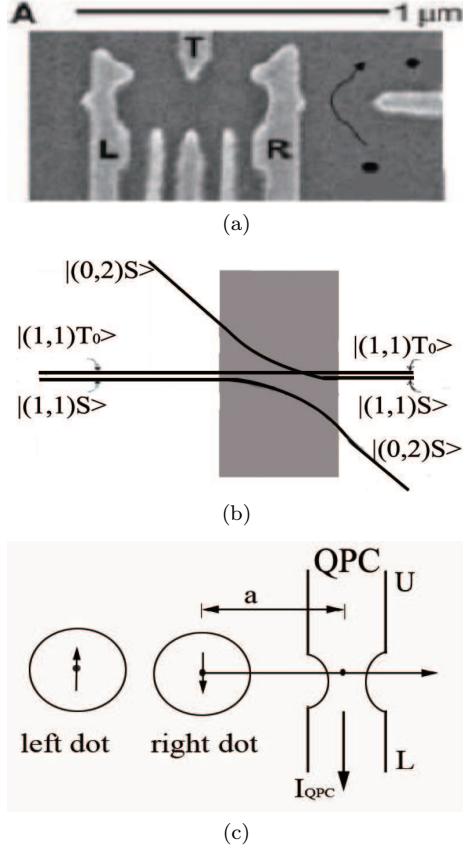
(a)

(b)

(c)

FIG. 1: (a) Micrograph of a sample of double quantum dots with a QPC in close proximity to the right dot. This figure is reproduced from Fig. 1 in [10]. (b) Lowest-energy states of the double quantum-dot system. (c) Demonstration of QPC near the double quantum-dot system. a gives the QPC position.

$E_T \approx E_S = 0$, therefore Eq. (1) is reduced to

$$\mathcal{H}_{DD} = -\varepsilon |(0,2)\,S\rangle\langle(0,2)\,S| + T_c(|(1,1)\,S\rangle\langle(0,2)\,S| + |(0,2)\,S\rangle\langle(1,1)\,S|).$$

The spin measurement relies on the spin-to-charge conversion. When an electron tunnels in or out of the right dot, the electrostatic potential in its vicinity is changed, and the current of QPC, I_{QPC} is very sensitive to the electrostatic changes correspondingly, thus the measurement of the changes of I_{QPC} reflects the number of electrons in the right dot. In the achieved experiment [10], the two electron spin states $|(1,1)\,S\rangle$ or $|(0,2)\,S\rangle$ can be measured with QPC since they have different charge distribution. Here we focus on the effect of spin states induced by QPC measurement, while the influence of the nuclear spins can be safely neglected in case of the imple-

mentation of spin-echo or dynamical nuclear polarization technology [10, 11].

The entire system would include the two quantum dots and the QPC near the right dot, and the whole system Hamiltonian can be written as

$$\mathcal{H} = \mathcal{H}_{DD} + \mathcal{H}_{QPC} + \mathcal{H}_{int},$$
$$\mathcal{H}_{QPC} = \sum_U E_U a_U^\dagger a_U + \sum_L E_L a_L^\dagger a_L,$$
$$\mathcal{H}_{int} = \sum_{L,U} \{\Omega_{UL} |(1,1)\,S\rangle\langle(1,1)\,S| (a_L^\dagger a_U + a_U^\dagger a_L)$$
$$+ \delta\Omega_{UL} |(0,2)\,S\rangle\langle(0,2)\,S| (a_L^\dagger a_U + a_U^\dagger a_L)\}.$$

In the above equations, the terms can be easily understood: (1) The quantum dots in the measurement process is a two-state system where $|(0,2)\,S\rangle$ and $|(1,1)\,S\rangle$ correspond to the case that two electrons are both in the right dot or each electron resides in a dot. (2) QPC is described as a standard one dimensional noninteracting electron system where $a_U^\dagger(a_U)$ and $a_L^\dagger(a_L)$ are the creation (annihilation) operators in the upper and the lower of QPC. (3) Since the presence of an extra electron in the right dot results in an effective increase of the QPC barrier, the hopping amplitude in QPC can be represented as Ω_{UL} and $\delta\Omega_{UL}$ corresponding to $|(1,1)\,S\rangle$ and $|(0,2)\,S\rangle$ states in quantum dots, respectively.

It is known that the off-diagonal density matrix elements can be destroyed by interaction with the measurement device. We can derive the equation of density matrix from the many-body Schröinger equation [12] or the standard technique with a Born-Markov approximation [16]. For convenience, the temperature of the system is assumed to be zero. The result master equations for the entire system can be obtained:

$$\frac{d\rho_{11}^{(n)}}{dt} = -D'\rho_{11}^{(n)} + D'\rho_{11}^{(n-1)} + \frac{i}{\hbar}T_c(\rho_{12}^{(n)} - \rho_{21}^{(n)}), \quad (2)$$
$$\frac{d\rho_{22}^{(n)}}{dt} = -D\rho_{22}^{(n)} + D\rho_{22}^{(n-1)} - \frac{i}{\hbar}T_c(\rho_{12}^{(n)} - \rho_{21}^{(n)}), \quad (3)$$
$$\frac{d\rho_{12}^{(n)}}{dt} = \frac{i}{\hbar}\varepsilon\rho_{12}^{(n)} + \frac{i}{\hbar}T_c(\rho_{11}^{(n)} - \rho_{22}^{(n)}) - \frac{1}{2}(D' + D)\rho_{12}^{(n)}$$
$$+ (DD')^{1/2}\rho_{12}^{(n-1)}. \quad (4)$$

Here $\rho_{11}(t)$ and $\rho_{22}(t)$ are the probabilities of finding the electron in the state $|(0,2)\,S\rangle$ or $|(1,1)\,S\rangle$, $\rho_{12}(t)$

and $\rho_{21}^*(t)$ are the off-diagonal density-matrix elements, $\rho_{12}(t) = \rho_{21}^*(t)$. The definition $D = \mathcal{T}eV_d/h$ or $D' = \mathcal{T}'eV_d/h$ is the transition rate of electron hopping from the upper to the lower of QPC corresponding to the quantum dot states $|(1,1)S\rangle$ or $|(0,2)S\rangle$, respectively. The QPC transmission probability is $\mathcal{T}$ and eV_d is the voltage bias. The index n denotes the number of electrons hopping in the QPC at time t.

In order to determine the influence of the QPC on the measured system, we trace out the QPC states $\rho_{ij} = \sum_n \rho_{ij}^{(n)}(t)$ in Eqs. (2,3,4), thus obtain

$$\frac{d\rho_{11}}{dt} = \frac{i}{\hbar}T_c(\rho_{12} - \rho_{21}), \qquad (5)$$

$$\frac{d\rho_{22}}{dt} = \frac{i}{\hbar}T_c(\rho_{21} - \rho_{12}), \qquad (6)$$

$$\frac{d\rho_{12}}{dt} = \frac{i}{\hbar}\varepsilon\rho_{12} + \frac{i}{\hbar}T_c(\rho_{11} - \rho_{22}) - \Gamma_d\rho_{12}. \qquad (7)$$

The last term in the equation for the non-diagonal density matrix elements ρ_{12} generates the exponential damping of the non-diagonal density matrix element with the "dephasing" rate

$$\Gamma_d = \frac{1}{2}(\sqrt{D} - \sqrt{D'})^2 = (\sqrt{\mathcal{T}} - \sqrt{\mathcal{T}'})^2\frac{eV_d}{4\pi\hbar}. \qquad (8)$$

From the definition of dephasing time T_2, we know that without any other decoherence source, $T_2 = 1/\Gamma_d$. And we can find that $\rho_{12} \to 0$ for $t \to \infty$. i.e. QPC measurement destroy spin coherence of the system.

Measurement effects. For studying the dephasing by measurement, we first start with the case of absence of QPC. Solving the Eqs. (5,6,7) for $\Gamma_d = 0$ with the initial conditions $\rho_{22}(0) = 1$ and $\rho_{11}(0) = \rho_{12}(0) = 0$ we obtain $\rho_{22}(t) = \frac{T_c^2 \cos^2(\omega t) + \varepsilon^2/4}{T_c^2 + \varepsilon^2/4}$, where $\omega = (T_c^2 + \varepsilon^2/4)^{1/2}$. This is an ordinary Bloch type time evolution of spin states in the quantum dots.

Then the measurement device QPC is in present, the Fig. 2a shows the time-dependence of the probability to find double-electron spin state in $|(1,1)S\rangle$, as obtained from the solution of the Eqs. (5,6,7) with the initial conditions $\rho_{22}(0) = 1$ and $\rho_{11}(0) = \rho_{12}(0) = 0$ for different cases: $\varepsilon = 3T_c$, and $\Gamma_d = 0$ (solid line), $\Gamma_d = T_c$ (dot line), and $\Gamma_d = 4T_c$ (dot-dashed line). It is clear that for small t the electron spins will be more localized in its initial state $|(1,1)S\rangle$ with the increase of Γ_d. Since the

measurement time Δt is inversely proportional to Γ_d, one could expect that the enhanced QPC measurement will slow down the rate of transition from spin state $|(1,1)S\rangle$ to $|(0,2)S\rangle$, i.e., the frequent repeated measurement will trap the system for small t. Therefore for small t our results seem to be in an agreement with quantum Zeno effect.
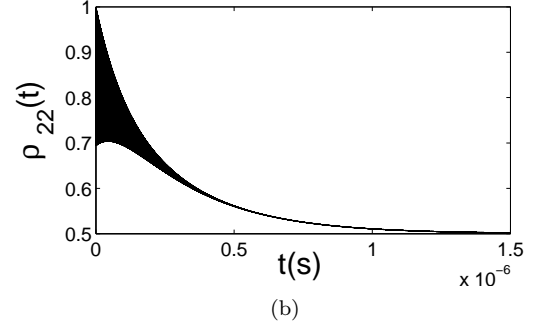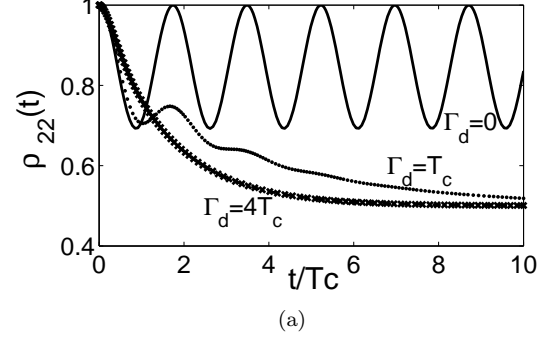

FIG. 2: (a) The time-dependence of the probability to find the double-electron state is $|(0,2)S\rangle$ for $\varepsilon = 3T_c$. The curves correspond to different values of the measurement induced dephasing rate: $\Gamma_d = 0$ (solid line), $\Gamma_d = T_c$ (dot line), and $\Gamma_d = 4T_c$ (dot-dashed line). (b) The time-dependence of the probability to find the double-electron state is $|(1,1)S\rangle$ for real experimental parameters.

Moreover, we would like to determine the spin evolution in the presence of QPC with real experimental parameters. To proceed, we consider for simplicity a δ-potential tunnel barrier $\Omega_{UL}(x) = \frac{\hbar^2 b}{m^*}\delta(x)$ for the QPC, and an additional Coulomb interaction as $\delta\Omega_{UL} = \Omega_{UL}(x) + \frac{e^2}{4\pi\epsilon_0 a}$ due to the quantum dot transformation from spin state $|(1,1)S\rangle$ to $|(0,2)S\rangle$. Using the standard scattering method the transmission change $\Delta\mathcal{T}$ for different spin states can be estimated as

$$\Delta\mathcal{T} = \mathcal{T} - \mathcal{T}' \approx \frac{e^2}{4\pi\epsilon_0 a}\frac{\mathcal{T}(1-\mathcal{T})}{E}\bigg|_{E_F}. \qquad (9)$$

By inserting typical experimental numbers in Eq. (9,8),

$\mathcal{T}(E = E_F) = 1/2$, $E_F = 10\,\text{meV}$, $a = 200\,\text{nm}$, $\epsilon = 13$ and $V_d = 1\,\text{mV}$, we obtain $\Delta\mathcal{T}/\mathcal{T} \approx 0.0277$ and $\Gamma_d \approx 1.139 \times 10^7\,\text{s}^{-1}$. Then we can calculate the time-dependence relation of $\rho_{11}(t)$ as shown in Fig. 2b for the experimental regime: $\varepsilon \approx 30\,\mu\text{eV}$, $T_c \approx 10\,\mu\text{eV}$. From Fig. 2b we can observe that the dephasing time is about 1 μs when the decoherence is only induced by the QPC measurement.

Since $\Delta\mathcal{T}$ is much smaller than the $\mathcal{T}$, we can expand the Eq. (8) as $\Gamma_d = (\sqrt{\mathcal{T}} - \sqrt{\mathcal{T}'})^2 \frac{eV_d}{4\pi\hbar} \approx \frac{eV_d(\Delta\mathcal{T})^2}{16\pi\hbar\mathcal{T}}$. Based on the Eq. (9), we can find that the dephasing time $T_2 = 1/\Gamma_d \propto 1/\Delta\mathcal{T}^2 \propto a^2$, and $T_2 = 1/\Gamma_d \propto 1/V_d$. It is clear that the QPC position a and the voltage bias V_d determine the dephasing time, so we can adjust Γ_d or T_2 by changing parameters a and V_d. The dephasing time from QPC measurement will extend with the decrease of V_d or the increase of a.

However, for real experimental systems when nuclear spins are not efficiently suppressed, we can include them phenomenologically the rate $1/T_2^{env}$ which describes the intrinsic decoherence in the dot, contributing to $1/T_2$. The contribution of QPC measurement to $1/T_2$ is calculated above as Γ_d, then the total decoherence rate is $1/T_2 = \Gamma_d + 1/T_2^{env}$. Since $\Gamma_d \approx 1.137 \times 10^7\,\text{s}^{-1}$ calculated from Eq. (8) and environment induced decoherence time $T_2^{env} \simeq 10$ ns which recently has been measured in a double-dot setup for singlet-triplet decoherence [10], we find that for a real experimental system in case the nuclear spins are not efficiently suppressed we have $1/T_2^{env} \gg \Gamma_d$, and the effects of QPC measurement may not be observed. Nevertheless, when the spin-echo or dynamical nuclear polarization technology are applied [10, 11] or the system is SiGe or graphene quantum dots [14, 15], the influence of nuclear spin is suppressed or completely eliminated. Thus the $1/T_2^{env}$ will be much small, the effect of the QPC measurement would dominate the decoherence of the system.

Conclusion. In summary, in this work we study in detail the effects generated by QPC measurement of two-electron spin states in double quantum-dot system. We give an effective Hamiltonian and derive the master equations of the whole system. Then we calculate the time evolution of spin states and find QPC measurement induced dephasing time $T_2 \approx 1\,\mu$s. We also provide a simple and transparent description of the enhanced QPC measurement which could trap the system for small t and be interpreted in terms of quantum Zeno effect.

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